
SON, THERE ARE TWO KINDS OF PEOPLE IN THIS WORLD...THOSE WHO ARE GOOD AT MATH, THOSE WHO ARE GOOD AT ENGLISH, AND THOSE WHO AIN'T GOOD AT NEITHER...

$$
\begin{array}{r|l}
 & \;\;71257 \\ \hline
5 & \;\;356285 \\
 & \;\;\underline{35} \\
 & \;\;\;\;06 \\
 & \;\;\;\;\;\underline{\;5} \\
 & \;\;\;\;\;\;12 \\
 & \;\;\;\;\;\;\underline{10} \\
 & \;\;\;\;\;\;\;28 \\
 & \;\;\;\;\;\;\;\underline{25} \\
 & \;\;\;\;\;\;\;\;35
\end{array}
$$

$\cancel{2}$

$\cancel{3}$

$\cancel{4}$

$\cancel{5}$

$\cancel{6}$

$\cancel{7}$

$\cancel{8}$

$\cancel{9}$

$\cancel{10}$

$\cancel{11}$ $\cancel{13}$

# 3.4 Zeros of Polynomial Functions

We know that an *n*th-degree polynomial can have at most n real zeros.

Now, in the complex number system, this statement can be improved.

That is, in the complex number system, every *n*th polynomial function has *precisely n zeros.*

**Fundamental Theorem of Algebra.**
If $f(x)$ is a polynomial function of degree $n$, where $n > 0$, then $f$ has at least one zero in the complex number system.

**Linear Factorization Theorem**.
If $f(x)$ is a polynomial function of degree $n$, where $n > 0$, then $f$ has precisely $n$ linear factors
$f(x) = an(x - c_1)\,(x - c_2)\ldots\,(x - c_n)$
where $c_1, c_2, \ldots, c_n$ are complex zeros.

a) $f(x) = x - 2$

b) $f(x) = x^2 - 6x + 9$

$(x-3)(x-3)$

c) $f(x) = x^3 + 4x$

$x(x^2+4)$

$x(x+2i)(x-2i)$

$x^2+4=0$

$x^2=-4$

$x=\pm 2i$

d) $f(x) = x^4 - 1$

$(x^2+1)\underbrace{(x^2-1)}$

$(x+i)(x-i)(x-1)(x+1)$

# Rational Zero Test

If a polynomial has integer coeffecients, then every rational zero of the polynomial has the form:

$$\text{zero} = \frac{p}{q}$$

where p and q have no common factors and,

p = a factor of the constant term
q = a factor of the leading coefficient

Possible rational zeros:

$$\frac{\text{factors of constant}}{\text{factors of leading coeffecient}}$$

Find the rational zeros of:

$$f(x) = x^3 + 2x^2 - x - 2$$

$p = \pm 1, 2$

$q = \pm 1$

possible rational zeros: $\pm 1, 2$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -1 & -2 \\ & \downarrow & 1 & 3 & 2 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$x^2 + 3x + 2$

$(x+2)(x+1)$

$x = 1, -1, -2$

$\boxed{(x-1)(x+2)(x+1)}$

Find the rational zeros of:

$f(x) = x^4 - x^3 + x^2 - 3x - 6$

$p = \pm 1, 2, 3, 6$

$q = \pm 1$

possible rational zeros: $\pm 1, 2, 3, 6$

$x = -1, 2$

| | | | | | |
|---|---|---|---|---|---|
| $-1$ | 1 | $-1$ | 1 | $-3$ | $-6$ |
| | | $-1$ | 2 | $-3$ | 6 |
| | 1 | $-2$ | 3 | $-6$ | 0 |

$(x^3 - 2x^2) + (3x - 6)$

$x^2(x-2) + 3(x-2)$

$(x-2)(x^2+3)$

$x = \pm i\sqrt{3}$

$(x+1)(x-2)(x - i\sqrt{3})(x + i\sqrt{3})$

Find the rational zeros of:

$f(x) = 2x^3 + 3x^2 - 8x + 3$

$p = \pm\ 1, 3$

$q = \pm\ 1, 2$

p.z. $= \pm\ 1, 3, \frac{1}{2}, \frac{3}{2}$

$8x^3 + 2x^2 - 5x \quad - 24$

$p = \pm 1, 2, 3, 4, 6, 8, 12, 24$

$q = \pm \ 1, 2, 4, 8$

$p.z = \pm 1, 2, 3, 4, 6, 8, 12, 24, \ \frac{1}{2}, \ \frac{1}{4}, \ \frac{3}{4}, \ \frac{3}{2},$

**Conjugate Pairs**

$(3+i)(3-i)$ $\qquad$ $(2-5i)$ $(2+5i)$

If $f$ is a polynomial function with real coefficients, then whenever $a + bi$ is a zero of $f$, $a - bi$ is also a zero of $f$.

Find a fourth degree polynomial with real coefficients that has -1, -1, and 3i as zeros

$-3i$

$(x+1)(x+1)(x-3i)(x+3i)$

$(x^2+2x+1)(x^2+9)$

$x^4+9x^2+2x^3+18x+x^2+9$

$\boxed{x^4+2x^3+10x^2+18x+9}$

Find all the zeros of:

$f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$

given that $1 + 3i$ is a zero

$1 - 3i$

$(x-(1+3i))(x-(1-3i))$

$(x-1-3i)(x-1+3i)$

$\cancel{x^2} - \cancel{x} + \cancel{3xi} - \cancel{x} + 1 - \cancel{3i} - \cancel{3xi} + \cancel{3i} - 9i^2$

$x^2 - 2x + 10$

$$\begin{array}{r} x^2 - x - 6 \\ x^2 - 2x + 10 \overline{\smash{)}\, x^4 - 3x^3 + 6x^2 + 2x - 60} \\ -\ \underline{x^4 - 2x^3 + 10x^2} \\ -x^3 - 4x^2 + 2x \\ -\ \underline{-x^3 + 2x^2 - 10x} \\ -6x^2 + 12x - 60 \\ \underline{-6x^2 + 12x - 60} \\ 0 \end{array}$$

$(x^2 - x - 6)$

$(x-3)(x+2)$

$$\boxed{x = 3, -2, 1+3i, 1-3i}$$

Use the given zero to find all the zeros of the function:

$$f(x) = 2x^3 + 3x^2 + 50x + 75$$

zero = 5i , -5i

$(x-5i)(x+5i) = x^2+25$

$$\begin{array}{r|l} & 2x \quad +3 \\ x^2+0x+25 & 2x^3+3x^2+50x+75 \\ & 2x^3+0x^2+50x \\ \hline & 3x^2+0x+75 \\ & 3x^2+0x+75 \\ \hline & 0 \end{array}$$

$$\boxed{x = 5i, -5i, -\frac{3}{2}}$$